

#### DIGITAL PULSE COMPRESSION TECHNIQUES

by

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Prepared Under:
Office of Naval Research
Contract No. N00014-67-A-0251-0023

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DIGITAL SYSTEMS LABORATORY Technical Report No. 77-12  Technical Report No. 77-12
<u>Title</u>
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Approved: V· N· Marinos P. N. Marinos Principal Investigator

Prepared under: Office of Naval Research

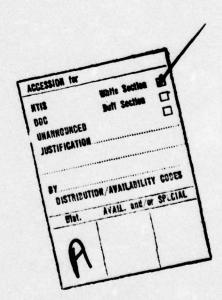
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#### ABSTRACT

The purpose of this work was to identify and study pulse compression techniques useful in LPI (i.e., Low Proabability of Intercept) applications. More specifically, the technique proposed should be capable of compression ratios of from 128 to 1024 or higher, and integrated sidelobes of -35 to -30 db throughout the compression range; furthermore, it should be easy to mechanize, and should result in minimal S/N loss. It was concluded that nonlinear FM is the pulse-compression scheme capable of satisfying all these requirements, and existing technologies make its mechanization feasible.



#### **ACKNOWLEDGMENTS**

The author wishes to thank Dr. W. M. Waters of the Naval Research Laboratory for the many helpful technical discussions concerning the problem, and acknowledges the support provided through the Office of Naval Research.

Thanks are also due to Mrs. Joyce Pate for typing the manuscript, and to Mr. Ben Rimer for his contributions to the computer programming effort.

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1. Nonlinear FM Pulse Compression System

## I. INTRODUCTION

The probability of detection of a radar signal in the presence of white Gaussian noise is maximized if the receiver is a matched filter to the signal [1]. In this case the receiver response becomes the signal autocorrelation function. In certain radar applications, it is also necessary that the transmitted waveform be rectangular and have as low an amplitude as possible but always consistent with the requirements for range resolution and probability of detection. These signal design constraints are best satisfied by using pulse compression techniques which will result in the minimal possible signal-to-noise power loss.

Nonlinear FM signals have relatively low time sidelobes and result in no loss in signal-to-noise ratio for as long as the receiver is matched to the transmitted nonlinear FM waveform. The compressed pulse, however, is highly sensitive to frequency variations, and becomes extremely distorted and develops high sidelobes as the doppler frequency is increased.

Since nonlinear FM suffers no signal-to-noise ratio loss under perfect match conditions, it becomes a serious candidate for use in a pulse compression system. In the sequel, a digital pulse-compression system is proposed based on nonlinear FM and capable of operating over an arbitrarily wide band of doppler frequencies without violating the requirements of specified sidelobe level and signal-to-noise ratio degradation.

## II. NONLINEAR FM PULSE COMPRESSION SIGNALS

Consider a narrow band signal s(t) centered at a carrier frequency  $f_c$  with an envelope a(t), and a phase modulation given by  $\phi(t)$ . That is,

$$s(t) = a(t)\cos[2\pi f_c t + \phi(t)], -T/2 \le t \le T/2$$

$$= 0, \text{ elsewhere}$$
(1)

where T is the pulse length. The equivalent complex representation of the modulation is given by

$$\widetilde{a}(t) = a(t) \exp[j\phi(t)]$$
 (2)

and its corresponding Fourier transform is expressed as

$$\widetilde{A}(f) = \int_{-\infty}^{\infty} a(t) \exp[j\{\phi(t) - 2\pi f t\}]dt$$

$$= A(f) \exp[j\theta(f)]$$
(3)

where A(f) denotes the modulus of  $\widetilde{A}(f)$ . In a similar manner, one may also write

$$a(t) = \int_{-\infty}^{\infty} A(f) \exp[j\{\theta(f) + 2\pi f t\}] df$$

$$= a(t) \exp[j\phi(t)]$$
(3a)

where a(t) is the modulus of  $\tilde{a}(t)$ .

The moduli a(t) and A(f) are related via Parseval's theorem which is analytically expressed by

$$\int_{-\infty}^{\infty} a^2(t)dt = \int_{-\infty}^{\infty} A^2(f)df = 2E$$
 (4)

where E is the energy in the real signal.

The signal autocorrelation function, z(t), which appears at the output of the matched filter receiver, is given by

$$z(t) = \int_{-\infty}^{\infty} \widetilde{a}(\tau) \ \widetilde{a}^*(t+\tau) \ d\tau$$
 (5)

where  $\tilde{a}^*(t)$  is the complex conjugate of  $\tilde{a}(t)$ . Using the Fourier transform of  $\tilde{a}(\tau)$ , one may rewrite Eq. (5) as

$$z(t) = \int_{-\infty}^{\infty} A^{2}(f) \exp[j2\pi ft] df$$
 (6)

or

$$A(f) = \left\{ \int_{-\infty}^{\infty} z(t) \exp[-j2\pi ft] dt \right\}^{1/2}$$
 (7)

The general problem which includes the nonlinear FM case as a special case may be stated as follows:

"Given the two moduli a(t) and A(f), is it possible to obtain a pair of phase functions  $\phi(t)$  and  $\theta(f)$  such that equation (3) (or 3a) is satisfied?"

With the exception of very few well-known cases, the general exact solution to this problem for arbitrary functions a(t) and A(f) is not known. This stems from the fact that Fourier theory does not provide a general set of necessary and sufficient conditions which must be satisfied by the moduli of a potential Fourier transform pair, that is, by a(t) and A(f) in this case. (Note, once again, that the reference here is to a(t) and A(f) and not to a(t) and a(f).) Nevertheless, one may still insist on an approximate solution and accept it or reject it depending on how well it satisfies expressions (3) and (3a). Such approximate solutions may be pursued by making use of the stationary phase method of approximate integration [2]. This method may be applied either to expression (3) or (3a).

According to the principle of stationary phase, the major contribution to the integral of a rapidly oscillating function will result from regions over which the phase of the integrand functions is "stationary"; that is, from regions over which the derivative of the phase function is zero.

Applying this technique to expression (3), one has

$$\frac{d}{dt} \left[ \phi(t) - 2\pi f t \right] \equiv 0$$
or
$$\phi'(t) = 2\pi f$$
(8)

Expression (8) implies that the points in time, t, at which the total phase becomes "stationary" will depend on the frequency f.

One obtains a similar relationship using expression (3a). That is,

$$\frac{d}{df} \left[ \theta(f) + 2\pi f t \right] = 0$$
or
$$\theta'(f) = -2\pi t$$
(9)

Expression (9) implies that the frequencies, f, at which the total phase becomes "stationary" will depend on the time, t.

It becomes apparent that in order to obtain  $\phi(t)$ , it is necessary that we determine the functional dependence of f on t, or

$$f = g(t) \tag{9a}$$

Use of (9a) in (8) and subsequent integration will provide the phase  $\phi(t)$ .

Similarly, one may use the inverse of (9a), that is,

$$t = g^{-1}(f) \tag{9b}$$

and expression (9) to determine  $\theta(f)$ .

Determining the functional dependence expressed by (9a) or (9b) is the ultimate objective of the stationary phase approximation method. This is illustrated by considering expression (3a) and expanding its phase function in a 3-term Taylor's series about the stationary point, say,  $f = \varepsilon$ , and performing the indicated operations. Key et al. [3] have shown that this yields the approximate expression

$$\widetilde{\mathbf{a}}(t) \cong \sqrt{2\pi} \frac{\mathbf{A}(\varepsilon)}{\sqrt{|\theta^{u}(\varepsilon)|}} \exp[\mathbf{j}\{2\pi\varepsilon t + \theta(\varepsilon) + \pi/4\}]$$
 (10)

where (+) is associated with  $\theta''(\epsilon) > 0$  and (-) with  $\theta''(\epsilon) < 0$ . Expression (10) for a(t) may be separated into the modulus and phase functions as follows:

$$a(t) = \sqrt{2\pi} \frac{A(f)}{\sqrt{|\theta^{H}(f)|}}$$
 (11)

and

$$\phi(t) = 2\pi f t + \theta(f) + \pi/4 \tag{12}$$

where  $\varepsilon$ , denoting the stationary point in frequency, has been replaced by f for clarity and in being consistent with expressions (8) and (9).

Differentiation of expression (9) and subsequent substitution in (11) yields  $a^{2}(t) dt = A^{2}(f) df \qquad (13)$ 

which in a sense establishes in differntial form the dependence of f on t or vice versa. Furthermore, expression (13) prescribes the manner in which the moduli a(t) and  $A(\bar{t})$  enter in this dependence.

According to Parseval's theorem (see expression (4)), Eq. (13) must yield

$$\int_{-\infty}^{\infty} a^2(t) dt = \int_{-\infty}^{\infty} A^2(f) df$$
 (14)

But more importantly, given time t (or frequency f) we want to know the corresponding frequency f (or time t) such that

$$\int_{-\infty}^{t} a^2(y) dy = \int_{-\infty}^{f} A^2(x) dx$$
 (15)

or equally valid

$$\int_{-\infty}^{t} a^{2}(y)dy = \int_{f}^{\infty} A^{2}(x)dx$$
 (16)

Equations (15) and (16) provide two possible solutions regarding the dependence of f on t, or vice versa, given two moduli functions a(y) and a(x). If the integrations implied by (15) and (16) are not tractable analytically, one may employ computational techniques to obtain the dependence of t on f or vice versa.

One special case of interest is the one in which the signal envelope or modulus a(t) is given by

$$a(t) = 1, -T/2 \le t \le T/2$$
  
= 0, elsewhere

and the spectral function or modulus A(f) is such that when processed by the matched filter receiver results in the prescribed response. The choice of A(f) for a specified sidelobe level and compressed pulse shape can be made from any of several well-known families of spectral functions [3,4].

Appendix-1 provides the computer program used to compute numerically the required phase function  $\phi(t)$  given  $a^2(x) = 1$ ,  $-T/2 \le t \le T/2$ , and arbitrary spectral functions for  $A^2(f)$ . In the program, use of expression (15) is made to compute the value of f corresponding to a given t, and this information is subsequently utilized in Eq. (8) to evaluate the derivative  $\phi'(t)$  at that point in time. From the numerically established behavior of the derivative, the program proceeds and evaluates  $\phi(t)$  which is the phase to be assigned to the transmitted signal.

## III. NONLINEAR FM SIGNALS IN THE PRESENCE OF DOPPLER SHIFT

The compressed pulse resulting from nonlinear FM pulse-compression systems is highly sensitive to frequency shifts due to doppler. This is due to a resulting phase mismatch condition which is responsible for causing severe distortions in the receiver response both in terms of degradation in the main lobe and the level of the range sidelobes.

The received spectrum is given by

$$S(\omega) = A (\omega + \omega_d) \exp[j\theta(\omega + \omega_d)]$$
 (17)

where  $\omega_{\bf d}$  =  $2\pi f_{\bf d}$ , the doppler frequency shift in rad/sec. The matched-filter transfer function is given by

$$H(\omega) = A(\omega) \exp[-j\theta(\omega)]$$
 (18)

and the output of the matched filter by

$$G(\omega) = A(\omega + \omega_d) A(\omega) \exp[j\{\theta(\omega + \omega_d) - \theta(\omega)\}]$$
 (19)

The function  $\theta(\omega)$  is obtained using procedures developed in the preceding section.

In order to overcome the detrimental effects of phase mismatch due to doppler frequency shifts, it is proposed that a bank of filters be designed which are perfectly matched in phase to signals received at equally spaced frequencies spanning the entire interval of doppler frequencies of interest (i.e.,  $f_c^{-f} d_{max}^{to} f_c^{+f} d_{max}^{to}$ ).

The center frequencies of the matched filters used in adjacent doppler channels are so spaced that the resulting deterioration in range sidelobe levels is kept within prespecified bounds.

The design of matched filters for processing nonlinear FM signals is a very difficult technical undertaking, and the task becomes even more complex if

the filters are required to process a wide variety of spectral functions, A(f). Such a need may arise either from technical or from tactical considerations, and it is best satisfied by implementing the matched filters and associated signal processing numerically. Such an approach requires that we design the matched-filter transfer function in a way which accounts for phase distortions due to doppler frequency shifts. Knowledge of the dependence of  $A(\omega)$  on  $\omega$  facilitates accounting of the doppler effects on the signal and its subsequent processing. In order to obtain the filter characteristics for perfect match in the presence of a doppler frequency shift equal to  $\omega_d$ , one must obtain  $\theta(\omega)$  from the moduli a(t) and  $A(\omega + \omega_d)$ . The modulus a(t) being the envelope of the transmitted signal is assumed to remain fixed, but  $A(\omega)$  is adjusted to reflect the frequency shift due to doppler.

The required spacing between the center frequencies of adjacent doppler filters for a specified sidelobe or signal-to-noise ratio deterioration is determined via numerical techniques utilizing the program given in Appendix-1. In the case of a square-root Taylor frequency spectrum, adjacent doppler channels with their center frequencies placed at points corresponding to a target velocity of 1.76 of MACH-1 result in a signal-to-noise ratio deterioration of less than -0.22 db; the corresponding distortion in the time response yields -18.1 db sidelobe level instead of the design value of -33.2 db.

Figure 1 offers a processing scheme for non-linear FM signal waveforms.

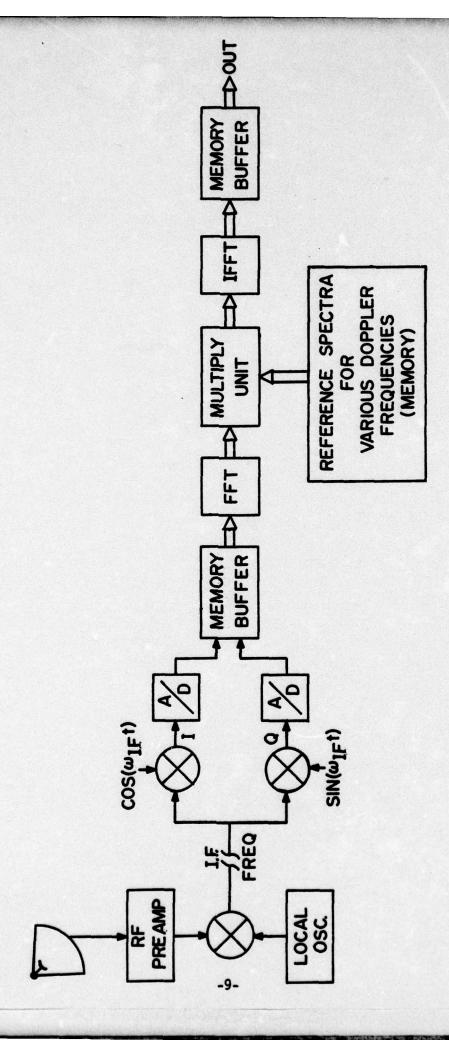


Fig. 1. - Nonlinear FM Pulse Compression System

- IV. DESIGN CONSIDERATIONS FOR THE NON-LINEAR FM PULSE COMPRESSION SYSTEM
  In order to accommodate information rates and signal waveforms compatible
  with modern radar system requirements, the design of the nonlinear FM
  pulse compression system must display
  - a/ flexibility in the choice of transfer function, and
  - b/ bandwidth capability of the order of many megahertzs.

The issue of system flexibility is easily settled by assuming digital implementation of the processor. This choice usually puts extra pressure on the system designer to develop FFT (Fast-Fourier Transform) units capable of coping with large bandwidths. In the entire system shown in Fig. 1, the FFT units are the only design challenges, and with present-day technology and existing FFT algorithms [5-12] their implementation has become both economically and technically feasible. The factors which determine the size of the FFT unit, and thus its cost, are the time-bandwidth product, speed and precision requirements of the system.

At this point in time, the design of FFT processors has reached sufficient maturity to satisfy most of the signal processing needs encountered in modern radar systems. The analysis and error sensitivity of various FFT algorithms have been well documented [13-20], and there is no need for their duplication here. It should be pointed out, however, that new advances in the area of parallel computational structures [21-25] point to more cost-effective FFT processor designs in the future, and thus to more economical and flexible signal processing systems.

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#### APPENDIX-1

The program given in this Appendix uses square-root Taylor weighting in the frequency domain for -40 db sidelobe level. The input quantities DEL1 and DELTA () specify the doppler frequency shift we wish to center the matched filter under design at, and the sequence of doppler frequencies for use in testing the performance of the matched filter, respectively. Note that DELTA or DEL1 are normalized quantities corresponding to the parameter,

$$\delta = (2 v/c)(f_c/\Delta f)$$

where  $\upsilon$  = target radial velocity; c = velocity of light;  $f_c$  = carrier frequency; and  $\Delta f$  = signal bandwidth. Furthermore, many of the parameters are assigned values within the program and not thru a READ statement.

ND = Number of doppler frequencies to be investigated

N = Number of sample points to be processed; M = N/16.

- NSTAGE = A positive integer such that N = 2 \*\* (NSTAGE). It denotes the number of FFT stages.
- AF() = Coefficients determining the polynomial signal taper employed. The illustrative values used in the program represent a -40 db sidelobe square-root Taylor weighting. AF() is DIMENSIONED to accommodate up to ten polynomial coefficients (see p. 20-18 of RADAR HANDBOOK by Skolnik.)
  - D = Compression ratio. The compression ratio is set equal to (N/4 0.25) which indicates four samples per cycle with the first sample at time t = 0, and the last sample omitted. (Also, D = COMPR)

MF = N/4

- MPSI = Number of Fourier coefficients wanted for approximating the signal phase function PSI() in the time domain.
- DEL1 = Center-frequency at which the nonlinear filter is tuned. It is given in normalized form and has been defined earlier in the Appendix.
- DELTA( ) = Doppler frequencies expressed in normalized form and in a manner similar to DEL1.

#### Discussion of subroutines and functions

- SUBROUTINE EVLPSI (NSTAGE, N, NN, D). It evaluates the Fourier coefficients B() for the approximation of the real signal phase function PSI(t).
- SUBROUTINE EVFLTR (NSTAGE, N, NN, D, FACTOR, DEL1). It evaluates the transfer function of the nonlinear filter tuned at DEL1. The parameter FACTOR is only defined and computed within the subroutine for latter use by other subroutines.
- SUBROUTINE RESPNS (NSTAGE, N, NN, D, DEL, FACTOR, XMAX). It evaluates
  the response of the nonlinear filter due to a unit-amplitude rectangular
  pulse of real phase PSI(t).
- 4. SUBROUTINE TSOPLT (MM). It provides a quick "line-printer" plot of the filter response (or of any function computed within the program).
- SUBROUTINE SAMPLE (NF, SIGN, D, DEL, NSTAGE). It samples the signal exp(JPSI(t))

- SUBROUTINE FFT(NSTAGE, SIGN). It evaluates the discrete Fourier transform of a set of sample points.
- SUBROUTINE TMPROC (NF, D, DEL, NSTAGE). It is used in conjunction with SUBROUTINE RESPNS()
- 8. SUBROUTINE SELECT (NF, VARBLE, OUTPUT). It selects between the functions SGNL() and TAPER() depending on the value NF.
- 9. SUBROUTINE INVERT (NF, XTOL, FTOL, NTOL, Y). It determines the root(s) of a function using Newton's method.
- 10. SUBROUTINE AMPLE (THIGH, N, NN). It samples the derivative function of PSI(t).
- 11. SUBROUTINE INTGRN(NF, C, E, Nf, NMAX, STOL, S). It integrates either function SGNL or TAPER depending on the value of NF.
- 12. FUNCTION PSI (D, DEL, SIGN, T). It provides the real phase of the rectangularly shaped signal assuming square-root Taylor weighting.
- 13. FUNCTION TAPER (PHI). It provides the spectral function of the signal which will result in the desired compressed pulse. In this case, use is made of square-root Taylor weighting.
- 14. FUNCTION SGNL(T). It provides a constant, unit-amplitude taper.

```
OPPLNEW JOB DU.DO7.AB]203, MARINOS, T=(2,00), P=50
DOPPLNEW EXEC WATFIV, REGION=220K
3YSIN DD DO7.AN3066/MARINOS, TIME=120, PAGES=50
COMPLEX X.DATANL
DOUGLE PRECISION D, DT, DFLOAT
DOUBLE PRECISION AF.XO.GXO.GCONST.WCONST
COMMON DT.X(2.2048).DATA(2048).TA(2048).A(101).B(101), MFOUR, MF
COMMON DATANL(2048).DATAX(2048)
COMMON AF(10).XO.GXO.GCONST.WCONST.NFLAG, IFLAG, M, MM
DIMENSION DELTA(8)
READ (1,2102) ND, NSTAGE, M, MPSI
READ (1,2103) DELTA (8)
READ (1,210
                             AF ( 4) =-0.0016105
AF ( 5) =0.0003474
N=2**NSTAGE
D=COMPP
          ME AD (1,102) (DELTA(K) , K=1,ND)
    102 FORMAT (8F10.0)
                              MFOUR=MPST
DI=N
WRITE
         WRITE (3.1)
CALL EVELTR (NSTAGE.N.NN.D.FACTOR.DEL1)
INITIALIZE THE LOOP FOR THE DELTA VALUES.
J=0
15 J=J+1
IF (J.GT.ND) GO TO 999
 IF (J.GT.ND) GO TO 999

OFL=DFLTA(J)

CALL PESPNS (NSTAGE.N.NN.D.DEL.FACTOR.XMAX)

WRITE (3.9) DI

WRITE (3.21) DEL

21 FOPMAT (3X.20H DELTA IS EQUAL TO .E15.4.//)

WRITE (3.898) DEL!

898 FOPMAT (13X.26HFILTER CENTERED AT DELTA= .E15.4.//)

WRITE (3.98) XMAX

98 FORMAT (3X.1THE SIGNAL TO NOISE PATIO IS EQUAL TO .E20.6.1 DBS.
   99 FORMAT (3x. THE AMPLITUDE OF THE RESPONSE IS EXPRESSED IN DRS. //)
CALL TSOPLT (MM)
WRITE (3.1)
GO TO 15
999 CONTINUE
                              END
```

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```
SUBROUTINE EVFLTR (NSTAGE,N.NN.D.FACTOR,DEL1)

COMPLEX X.DATANL

DOUPLE PRECISION D.DT.DFLOAT

DOUBLE PRECISION AF.XO.GXO.GCONST.WCONST

COMMON DT.X(2.204d),DATA(2048).TA(2048),A(101),B(101),MFOUR.MF

COMMON DATANL(2048),DATAX(2048)

COMMON AF(10),XO.GXO.GCONST.WCONST.NFLAG.IFLAG.M.MM

COMPUTE REQUIRED FILTER COMPENSATION. NEXT:

DT=D/DFLOAT(N-1)

LSTAGE=NSTAGE+1

MF1=MF+1

CALL SAMPLE (2.+1..D.O.,NSTAGE)

CALL FFT (LSTAGE.-1.)

DATANL(1)=X(2.1)

DO 250 K=2.MF

DATANL(N)=X(2.K)

DATANL(N)=X(2.K)

DATANL(NN+2-K)=X(2.NN+2-K)

SO CONTINUE

DO 251 K=MF1.N

DATANL(N+2-K)=CMPLX(0.0.0.0)

DATANL(N+2-K)=CMPLX(0.0.0.0)

DATANL(N+1)=CMPLX(0.0.0.0)
       251 CONTINUE

DATANL(N+1) = CMPLX(0.0.0.0)

CALL SAMPLE (2.+1..D.DEL1.NSTAGE)

CALL FFT (LSTAGE.-1.)

DATANL(1) = DATANL(1) / X(2.1)

DO 8001 K=2.MF

DATANL(K) = DATANL(K) / X(2.K)

DATANL(NN+2-K) = DATANL(NN+2-K) / X(2.NN+2-K)

CONTINUE
DATANL (NN+2-K) = DATANL (NN-2

)01 CONTINUE
WPITE (3.1)

1 FORMAT (||H|)
CALL SAMPLE (2.-1..)
DATANL (1) = DATANL (1) * X (2.1)

DO 8000 K = 2.MF

DATANL (1) = DATANL (1) * X (2.K)

DATANL (K) = DATANL (K) * X (2.K)

DATANL (MN+2-K) = DATANL (NN+2-K) * X (2.NN+2-K)

100 CONTINUE
THE COMPLEX ARPAY DATANL CONTAINS THE FILTER

DAT = REAL (DATANL (1)) * * 2. * A IMAG (DATANL (1)) * * 2

DO 8002 K = 2.MF

GAT = REAL (DATANL (K)) * * 2. * A IMAG (DATANL (K)) * * 2

DAT = DAT + 2. * GAT

DAT = DAT + 2. * GAT
         DAT=DAT+2.*GAT

DAT=DAT+2.*GAT

OZ CONTINUE
THE CONSTANT 'FACTOR' IS EQUAL TO THE DENOMINATOR IN THE DEFINI-
TION OF THE SIGNAL-TO-NOISE RATIO.

FACTOR=DAT/2.

PETURN
END
```

17

```
SUBPOUTINE RESPNS (NSTAGE.N.NN.D.DEL.FACTOR.XMAX)

COMPLEX X.DATANL

DOURLE PRECISION D.DT.DFLOAT

DOURLE PRECISION AF.XO.GXO.GCONST.WCONST

COMMON DT.X(2.2048).DATA(2048).TA(2048).A(101).B(101).MFOUR.MF

COMMON DATANL(2048).DATAX(2049)

COMMON AF(10).XO.GXO.GCONST.WCONST.NFLAG.IFLAG.M.MM

PREPARATION OF THE OUTPUT OF THE RESPONSE.

CALL TMPROC(2.D.DEL.NSTAGE)

DO 1 K=1.NN

DATA(K) = DATA(K)/FACTOR

TA(K) = -0.DFLOAT(K-1)*DT

11 CONTINUE

YMAX=DATA(1)

DO 97 I=1.NN

IF (DATA(I).GT.XMAX) XMAX=DATA(I)

97 CONTINUE

DO 12 K=1.MM

DATA(K)=DATA(N-M+K)

DATA(K)=TA(N-M+K)

TA(K)=TA(N-M+K)

12 CONTINUE

XMAX=10.**ALOG10(XMAX)

RETURN

END
```

```
SUBFOUTINE SAMPLE (NF.SIGN.D.DEL,NSTAGE)

COMPLEX X.DATANL

DOUBLE PRECISION D.DT

DOUBLE PPECISION AF.XO.GXO.GCONST.WCONST
COMMON DT.X(2.2048).DATA(2048).TA(2048).A(101).B(101).MFOUR.MF

COMMON AF(10).XO.GXO.GCONST.WCONST.NFLAG.IFLAG.M.MM

N=2**NSTAGE
NI=N+1
NM=2*N

IF (NF.EQ.1) GO TO 100

T=-D/2.-DT

DO ? I=1.N

T=T.DT

Y=PSI(D.DEL.SIGN.T)

YY=COS(Y)

YS=SIN(Y)

X(1.1)=CMPLX(VC.YS)

2 CONTINUE

DO 3 I=N1.NN

X(1.1)=CMPLX(0.0.0.0)

3 CONTINUE

PFILIDN

OO Y=DATA(1)

YC=COS(Y)

YS=SIN(Y)

X(1.1)=CMPLX(YC.YS)

DO 4 I=2.MF

Y=DATA(1)

YC=COS(Y)

YS=SIN(Y)

X(1.1)=CMPLX(YC.YS)

X(1.NN-Y-I)=X(1.1)

RETURN

END
```

```
SUBROUTINE FFT ( NSTAGE.SIGN)

COMPLEX X.DAIANL.W

DOUBLE PRECISION D.DT

DOUPLE PRECISION D.DT

DOUPLE PRECISION AF.XO.GXO.GCONST.WCONST

COMMON D.X.(2.249).DAIA(2048).TA(2048).A(101).B(101).MFOUR.MF

COMMON DATANL(2049).DAIAX(2048)

COMMON DATANL(2049).DAIAX(2048)

COMMON DAIANL(2049).DAIAX(2048)

INTEGER R

N=2**NSTAGE

N=2**NSTAGE

N=2**N/2

FL IN=11

PHI2N=6.2831853/FLTN

DO 3 J=1.NSTAGE

N2J=N/(2**J)

NP=N2J

NI=(2**J)/2

DO 2 I=1.NI

IN 2J=(1-1)*N2J

FL IN2J=(1-1)*N2J

FL IN2J=(1-1)*N2J

FL IN2J=(1-1)*N2J

ISUB=1NR

ISUB=*IN2J*

ISUB=*IN2
```

```
SUBROUTINE TMPROC (NF,D,DEL,NSTAGE)

COMPLEX X.DATANL

DOURLE PRECISION D.DT

DOUBLE PRECISION AF.XO.GXO.GCONST.WCONST

COMMON DT.X(?.2048).DATA(2048).TA(2048),A(101).B(101).MFOUR.MF

COMMON DATANL(2048).DATAX(2048)

COMMON AF(10).XO.GXO.GCONST.WCONST.NFLAG.IFLAG.M.MM

LSTAGE=NSTAGE+1

NN=2**LSTAGE

CALL SAMPLE (NF.+1..D.DEL.NSTAGE)

CALL SAMPLE (NF.+1..D.DEL.NSTAGE)

CALL FFT ( LSTAGE.-1.)

DO 2 K=1.NN

X(1.K)=X(2.K)*DATANL(K)

2 CONTINUE

CALL FFT ( LSTAGE.+1.)

RETURN
END
```

```
FUNCTION PSI (D,DEL,SIGN,T)

COMPLEX x.DATANL

DOUPLE PRECISION D.DT

DOUPLE PRECISION AF.x0.GXU.GCONST.wCONST

COMMON DI.X(2.2048).DATA(2048).TA(2048),A(101).B(101).MFOUR.MF

COMMON DATANL(2048).DATAX(2048)

COMMON AF(10).Y0.GX0.GCONST.WCONST.NFLAG.IFLAG.M.MM

DIMENSION CX(101)

PI=3.1415927

Y=2.*PI*T/D

CX(1)=CX1

CX(2)=2.*CX1*CX1-1.

DO 2 L=3.MFOUR

CX(L)=2.*CX(L-1)*CX1-CX(L-2)

2 CONTINUE

Z=0.

DO 3 L=1.MFOUP

Z=Z-B(L)*CX(L)/FLOAT(L)

3 CONTINUE

Z=D*Z

E-DI*T*T*D
                       Z=D*Z
E=PI*T*T/D
                     Z=Z+E
Z=Z+E
E=2.*PI*DEL*T
G=SIGN*Z
PSI=G+E
RETURN
END
```

```
SUBROUTINE TSOPLT (N)

COMPLEX X.DATANL

DOUBLE PRECISION D.DT

DOURLE PRECISION AF.X0.GX0.GCONST.WCONST

COMMON DT.X(2.2048).DATA(2048).TA(2048),A(101).B(101).MFOUR,MF

COMMON DATANL(2048),DATAX(2048)

COMMON AF(10).X0.GX0.GCONST.WCONST.NFLAG.IFLAG.M.MM

DATA ISTAP/IH*/
          XMAX=DATA(1)
         XMAX=DATA(1)

XMIN=DATA(1)

DO 1 J=1.N

IF (DATA(I).GT.XMAX) XMAX=DATA(I)

IF (DATA(I).LT.XMIN) XMIN=DATA(I)

CONTINUE

DIFF=XMAX-XMIN
RETURN .
```

```
SUBROUTINE EVLPSI (NSTAGE,N,NN,D)

COMPLEX X.DATA!!L

DOUBLE PRECISION D.DT

DOUBLE PRECISION THIGH.OUT

COMMON DIANAL (2048).DATAX (2048).TA (2048).A (101).B (101).MFOUR.MF

COMMON DATANL (2048).DATAX (2048)

COMMON DATANL (2048).DATAX (2048)

THIGHED/2.

LSTAGE=NSTAGF+1

EVALUATION OF THE CONSTANTS.

WPITE (3,1)

1 FORMAT (1H)

CALL INTGRN (1.0.D 00.THIGH.1.512.0.1D-05.OUT)

IF (NFLAG.GT.0) GO TO 990

GCONST=OUT

CALL INTGRN (2.0.D 00.0.5D 00.1.256.0.1D-05.OUT)

IF (NFLAG.GT.0) GO TO 990

WCONST=OUT

CALL INTGRN (2.0.D 00.0.5D 00.1.256.0.1D-05.OUT)

IF (INTGRN (2.0.D 00.0.5D 00.1.256.0.1D-05.OUT)

IF (IFLAG.GT.0) GO TO 990

WCONST=OUT

CALL INTGRN (2.0.D 00.0.5D 00.1.256.0.1D-05.OUT)

IF (IFLAG.GT.0) GO TO 990

WCONST=OUT

CALL FIT (LSTAGE.-1.)

DO 2 I=1.MFOUR

B (1)=-2.*AIMAG(X(2,I+1))

2 CONTINUE

DO 10 I=1.MFOUR.2

R (1)=-B(1)

10 CONTINUE

RETURN

990 WPITE (3.8)

B FORMAT (3X.*AN INTEGRATION IN SUBROUTINE EVLPSI HAS FAILED.*//)

END
```

```
SUBROUTINE SELECT (NF, VARBLE, OUTPUT)

COMPLEX X.DATANL

DOUBLE PRECISION D.DT

DOUBLE PRECISION AF.XO.GXO.GCONST.WCONST

DOUBLE PRECISION VAPBLE.OUTPUT.SGNL.TAPER

COMMON DT.X(2.2048).DATA(2048).TA(2048).A(101).B(101).MFOUR.MF

COMMON DATANL(2043).DATAX(2043)

COMMON AF(10).XO.GXO.GCONST.WCONST.NFLAG.IFLAG.M.MM

GO TO (10.20).NF

10 OUTPUT=SGNL(VAPBLE)

PETURN

20 OUTPUT=TAPER(VARBLE).

PETURN

END
```

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```
SUBROUTINE INTGRN (NF,C,E,N1,NMAX,STOL,S)

COMPLEX X.DATANL
DOUBLE PRECISION D.DT
DOUBLE PRECISION AF.X9.GX0.GCONSI,WCONST
DOUBLE PRECISION C.E.STOL.S.DFLOAT.S1.H.HALF.XY.DS.DABS.S2.FC.FE.

IMALF].CI
COMMON DATANL (2048).DATA (2048).TA (2048).A (101).B(101).WFOUR.MF
COMMON DATANL (2048).DATAX (2048)

COMMON AF (10).X0.GX0.GCONST.WCONST.NFLAG.IFLAG.M.MM
NFLAG=0
M1=N1
S1=0.0D 00
CALL SFLECT (NF.C.FC)
CALL SFLECT (NF.E.FE)

IM | 2.841
IF (M1.GT.NMAX) GO TO 999
H=(F-C)/DFLOAT(M1)
HOV2=H/2.
S=0.D 00
C1=C.+HOV2
CALL SFLECT (NF,C1.HALF)
NM1=M1-1
DO 2 1=1.NM1
XY=C.DFLOAT(I)*
XY=C.DFLOAT(I)*
XY=C.DFLOAT(I)*
XY=C.DFLOAT(I)*
XY=C.DFLOAT(I)*
CALL SFLECT (NF,C1.HALF1)

ALF =HALF +HALF1

2 CONTINUE
S=(H/A-)*(FC.4.*HALF+2.*S.*FE)
DS=S-S2
IF (DABS(DS).GT.STOL) GO TO 3

RETURN
END
```

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```
SUBPOUTINE INVERT (NF.XTOL.FTOL.NTOL.Y)

COMPLEX X.DATANL

DOUBLE PRECISION O.DT

DOUBLE PRECISION AF.XO.GXO.GCONST.WCONST

DOUBLE PRECISION AF.XO.GXO.GCONST.WCONST

COMMON DATA (20.48).DATA (20.48).A(101).B(101).MFOUP.MF

COMMON DATANL (20.43).DATAX (20.48).A(101).B(101).MFOUP.MF

COMMON AF.10).XO.GXO.GCONST.WCONST.NFLAG.IFLAG.M.MM

IFLAG=0

00 1 I=1.NTOL

FXO=GXO.Y

IF (DARS(IXO).LT.FTOL) RETURN

CALL SELECT (NF.XO.DERIV)

IF (DEPLY.EC.O.D 00) GO TO 999

DELTAX=FXO.DERIV

X1=XO

X0=XO.PCELTAX

CALL INTGPN (NF.X1.40.1.256.0.1D-05.GX1)

GXO=GXO.GX1

IF (DABS(DELTAX).LT.XTOL) RETUPN

1F (DABS (DELTAX).LT.XTOL) RETUPN

1F (DABS (DELTAX).GT.XTOL) GO TO 990

PETUPN

989 WRITE (3.2)

PETURN

999 WRITE (3.3)

3 FORMAT (3X.49H THE INTEGRATION 1N SUBROUTINE INVERT HAS FAILED ,

1//)

PFTURN

999 WRITE (3.3)

3 FORMAT (3X.45H THE DERIVATIVE IN SUBROUTINE INVERT IS ZERO .//)

1FLAG=1

990 WRITE (3.4)

4 FORMAT (3X.50H DELTAX IN SUBROUTINE INVERT IS GREATER THAN XTOL

1FLAG=1

PETURN

1FLAG=1

PETURN
```

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```
FUNCTION TAPER (PHI)

COMPLEX x, DATANL

DOUBLE PRECISION D,DT

DOUBLE PRECISION D12,CX,Y,PHI,CX1,Z,DLOG,DEXP,TAPER,DCOS

DOUBLE PRECISION AF.X0.GX0.GC0NST.wCONST

COMMON DT.y (2.2048).DATA(2048).TA(2048),A(101).B(101).MFOUR.MF

COMMON AF(10).X0.GX0.GCONST.wCONST,NFLAG.IFLAG.M.MM

DIMENSION CX(10)

PIZ=6.283185307179

Y=PIZ**PHI

CX1=DCOS(Y)

CX(1)=2.*CX1*CX1-1.

DO 2 L=3.5

CX(L)=2.*CX1*CX1-1.

DO 2 L=3.5

CX(L)=2.*CX(L-1)*CX1-CX(L-2)

CONTINUE

TAPER=7

RETURN

END
```

FUNCTION SGNL (T)

COMPLEX X.DATANL

DOUBLE PRECISION D.DT

DOUBLE PRECISION AF, X0, GX0, GCONST, WCONST

DOUBLE PRECISION T, SGNL

COMMON DT, X(2, 2048), DATA(2048), TA(2048), A(101), B(101), MFOUR, MF

COMMON DATANL (2048) DATAX(2048)

COMMON AF(10), X0, GX0, GCONST, WCONST, NFLAG, IFLAG, M, MM

SGNL=1.D 00

RETURN
END

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DIGITAL PULSE COMPRESSION TECHNI	IQUES					
4. DESCRIPTIVE NOTES (Typo of report and inclusive Final Report (2/1/74 - 10/1/76)	na <del>cana</del> j					
Peter N. Marinos						
6. REPORT DATE January 1977	72. TOTAL NO. OF PAGES	76. NO. OF REFS				
NOO14-67-A-0251-0023		Digital Systems Laboratory				
	SO. OTHER REPORT HOLES (Amplify) TR-77-1	y other numbers that may be assigned				
Distribution of this document is	s unlimited					
11. SUPPLEMENTARY NOTES	Office of Naval Res Requisition/Purchase	Office of Naval Research Requisition/Purchase Request No. NRL Req. 00173-4-006227, Rev. \$2/6-18-76				

The purpose of this work was to identify and study pulse compression techniques useful in LPI (i.e., Low Probability of Intercept) applications. More specifically, the technique proposed should be capable of compression ratios of from 128 to 1024 or higher, and integrated sidelobes of -35 to -30 db throughout the compression range; furthermore, it should be easy to mechanize, and should result in minimal S/N loss. It was concluded that nonlinear FM is the pulse-compression scheme capable of satisfying all these requirements, and existing technologies make its mechanization feasible.

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Nonlinear FM							
Fast Fourier Transform							
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